# Combinatorics II

## Discrete Algebraic Structures

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# Inclusion-exclusion principle

$$|A_1 \cup A_2 \cup \cdots \cup A_n| = \sum_{r=1}^{n} (-1)^{r+1} \sum_{1 \leq i_1 < i_2 < \cdots < i_r \leq n} |A_{i_1} \cap \cdots \cap A_{i_r}|$$

$\Rightarrow$ simple cases (should be sufficient for the exam):

$n = 2$:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$n = 3$:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

# Double counting

For $R \subseteq A \times B$:

$$\sum_{a \in A} \deg(a) = \sum_{b \in B} \deg(b)$$

Example application:

- prove $a \cdot m = b \cdot n$ by constructing suitable sets $A$ and $B$ (with known cardinality $|A| = a, |B| = b$) and relation $R$ (with knwon degrees $\deg(a) = m \ \forall a \in A$, $\deg(b) = n \ \forall b \in B$)

# Pigeonhole principle

For $|A| > k|B|$ and $f : A \to B$:
There exist $k+1$ distinct elements $x_1, x_2, \ldots, x_{k+1}$ such that

$$f(x_1) = f(x_2) = \cdots = f(x_{k+1})$$