Abstract Algebra II Discrete Algebraic Structures

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Homomorphisms

- $\diamond\,$ in general a structure preserving map
- bijective homomorphisms are called isomorphisms
- $\diamond \ f: G
 ightarrow H$ is a group homomorphism if for all $a, b \in G$
 - $\rightarrow f(a \cdot_G b) = f(a) \cdot_H f(b)$
- $\diamond\,$ the definition implies that
 - $ightarrow f(a^{-1}) = f(a)^{-1}$ ightarrow f(e_G) = e_H
- $\rightarrow\,$ those are in general easy to check to quickly rule out some functions

Rings

$$◊ we say × distributes over + if for all a, b, c ∈ R → a × (b + c) = a × b + a × c and (b + c) × a = b × a + c × a$$

- \diamond we call (R,+, imes) a ring, if
 - \diamond (*R*, +) is a commutative group with neutral element 0
 - \diamond (*R*, \times) is a monoid with neutral element 1
 - $\diamond \ 0 \neq 1$
 - $\diamond~\times$ distributes over +